Lab 1: Measurements and Experimental Errors (M1)

The goal of an experiment is usually to measure something, such as the mass of an object, the position of an object or the velocity of a projectile. When the measurement is finished you have new knowledge. However, **real measurements cannot be done without experimental error**. It could be very small, but it is always present. To be able to compare the results of different measurements with each other, or to compare them with a theoretical model (like the ones we will be discussing in class), it is essential that we have a way to **estimate the uncertainty, i.e., the experimental error**, associated with our measurements. In this lab, you will collect, analyze experimental data and calculate the experimental error.

This lab will also introduce you to the computerized, ultrasonic motion sensors that you will use in many of Physics 22000 experiments. You will use them to measure position of various objects.

Objectives:

- Become familiar with the operation and use of the motion sensors and the associated software.
- Examine motion by creating distance (position) vs. time graphs.
- Discover the relationship between position vs. time and velocity vs. time graphs.
- Learn about how to calculate experimental results using the average value and the sample standard deviation.

Equipment:

You have the following materials at your disposal.

- A computer equipped with the PASCO motion sensor hardware and software.

Descriptions of Motion

- **Position**: the location of an object with respect to the <u>origin</u> (the motion sensor is the origin ($x = 0$) from which distances are measured).

- **Average Velocity**: the displacement (change in position) of an object during an interval divided by the time that elapsed during that interval.

$$v = \frac{Displacement}{Elapsed\ Time} = \frac{x - x_0}{t - t_0} = \frac{\Delta x}{\Delta t}$$

   Velocity is a vector. It has <u>speed</u> (how fast something is moving - the length of the velocity vector) and <u>direction</u>. Average velocity for a constant speed is equal to the slope of the position as a function of time graph for that time interval. Therefore, for a faster velocity, the slope of the position vs. time graph is more inclined (has a larger slope) than that for a slower velocity.

- **Instantaneous velocity**: indicates how fast an object is moving and the direction of the motion at each instant of time.

$$v = \frac{\Delta x}{\Delta t} \quad \text{with } \Delta t \text{ extremely small}$$

<u>*Remember:*</u> instantaneous velocity does not always equal average velocity. For example, suppose a person takes a road trip in the car and travels the distance of 60 miles in one hour. The average velocity for that trip is 60 miles per hour. However, at a given instant in time, the instantaneous velocity could be 70 miles per hour, 50 miles per hour, -40 miles per hour or any other value, so long as the average of all the instantaneous velocities is 60 miles per hour.

   For one-dimensional motion, the sign (+ or -) of the velocity indicates the direction of the motion.

$$+v \ = \ \text{moving } \underline{\text{away}} \text{ from the origin (the motion sensor).}$$
$$-v \ = \ \text{moving } \underline{\text{towards}} \text{ the origin (the motion sensor).}$$

   The motion detector is the origin ($x = 0$) from which all distances are measured.

Procedure:

Activity 1: Introduction to Ultrasonic Motion Sensors

   The ultrasonic motion sensors work by sending out ultrasonic pulses and measuring the time between moments when a pulse is emitted and when its reflection returns to the sensor. The ultrasonic is identical to ordinary sound, except that its frequency is too large to be heard (we use 50,000 Hz in this lab). The motion sensor acts as a transmitter, when it sends out pulses, and as a receiver, when it listens for echoes. The software computes the delay Δt between sending the pulse and receiving the echo. Then, using the speed of sound in air (v_{sound} = 343 m/s at 20ºC = 68ºF; the speed of ultrasound is the same as the speed of audible sound waves), the software determines the distance $x = \frac{1}{2} * \Delta t * v_{sound}$ from the detector to the object that reflected the pulse. The factor ½ is there because the ultrasound pulse must travel the distance twice – from the sensor to the object and back. Usually, the distance is plotted as a function of time. Some animals, e.g., bats, use the same method of ultrasonic pulses to measure distances. Each time the motion sensor sends out an ultrasonic pulse it also makes an audible sound. This clicking sound is what you hear when the motion sensor is emitting pulses.

The same procedure is repeated 10 or 20 times per second. Therefore, it seems to be a continuous process. Each time the motion sensor sends out an ultrasonic pulse it also makes an audible sound ($f = 20$ Hz). This clicking sound is what you hear when the motion sensor is emitting pulses.

The ultrasonic motion sensor can give very accurate results, but it does have some limitations:

(1) The object must be large enough to give a good reflection.
(2) The detector will not correctly measure anything closer than 12 cm.
(3) The sensor must be aimed well, so that the reflected pulses will return to the sensor.
(4) The closest object in the line of sight will be the one that is "seen" by the ultrasonic sensor.
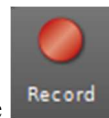
In Physics 22000 lab, computers will be used to collect and present data for most of the experiments. The software application used for these experiments is called ***Capstone*** and it was provided by ***PASCO Scientific***.

Login using your **ITaP** account (**Purdue career account**). Enter your login name and password.

Open "M1 Activity 1". The application program *Capstone* by PASCO Scientific will configure for *Activity 1*.

Now you are ready to record some motion. You are going to use ultrasonic sensor to measure position of the moving object as a function of time. The moving object is going to be you or your lab partner! By trial and error, determine how close you must be for the sensor to record your position. In some cases, holding a flat object like a book in front of the moving person leads to a better (smoother) position versus time curve. Flat objects reflect more ultrasound than fabric does. **Move the object towards and away from the sensor and try to match the pattern on the position versus time graph.**
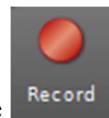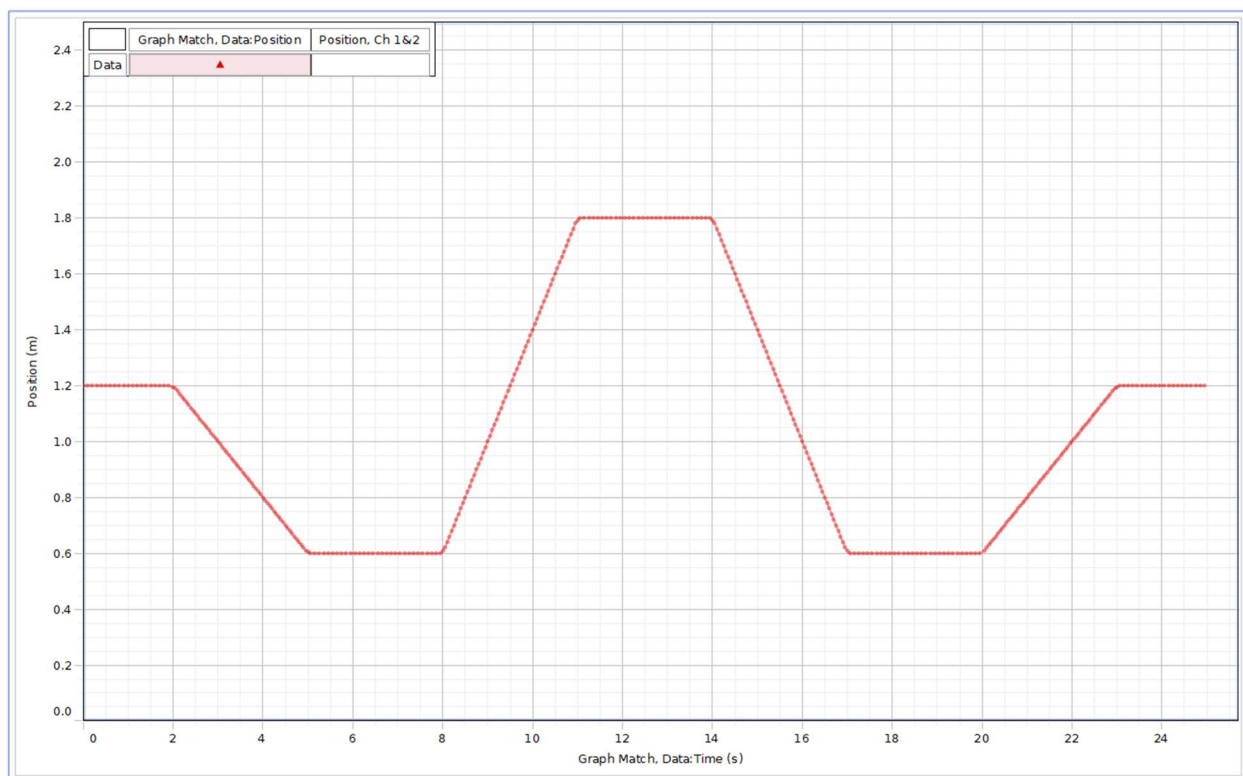
When you are ready to start collecting data, click on the **Record** button located below the graph. In most activities, the computer will stop collecting data automatically after the preset time (25 sec in this case).
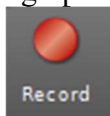
It usually takes a few attempts to get a reasonably good match. When you are happy with the result, print the graph by selecting "**Print**" from the "**File**" menu. Attach this printout to your lab report. **Do not save any changes!**

Activity 2: Prediction of Velocity from the Position vs. Time Graph.

Open the document for Activity 2.

You should see two graphs: the position versus time (top) and velocity versus time (bottom)

graphs. Click on the **Record** button to start data recording. Keep looking at the computer monitor and move to match the position vs. time graph. It is the same pattern as in Prelab Question 1.
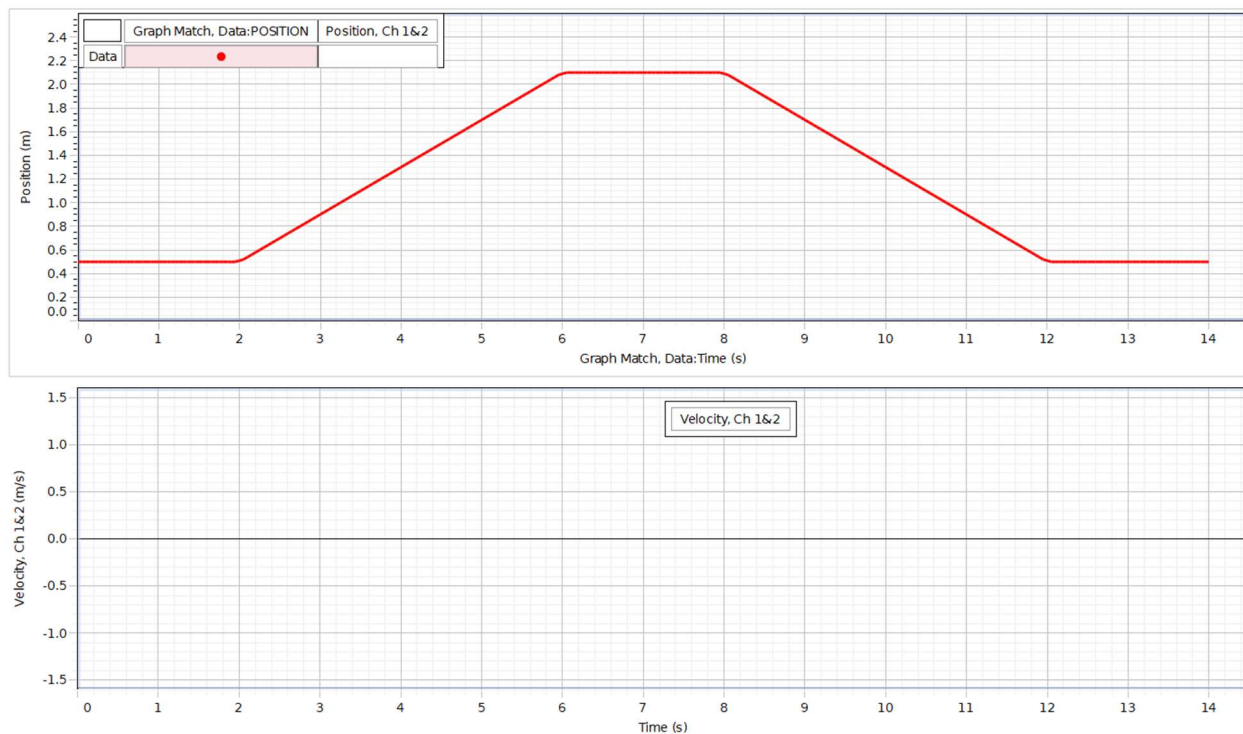
Using the position versus time data, computer will create the velocity versus time graph. Examine that graph and if you have a good match print it. If you do not match the provided pattern, simply try it again. Print a copy of your best run. Next, select **Print** from **File** menu. Label the graph: "Velocity vs. Time from a Position vs. Time Match".

Activity 3: Average Value and Standard Deviation

Experimental errors.

In most areas of science and technology, you must be familiar with taking measurements and calculating results based on those measurements. You must also be able to understand the accuracy of your measurements and determine the uncertainties in your results. In many experiments you ever perform (including this one), you will make multiple measurements of a quantity, and will then want to combine them to get a final value.

The key questions we want to learn how to handle are:

(a) How should we combine multiple measurements to obtain a single "best'" value?

(b) How should we estimate the uncertainty (or "experimental error") in this best value?

These questions arise in virtually all experiments or measurements, in physics and elsewhere.

In all measurements, the result you obtain is never exact. For example, in the present experiment you will measure the distance x between the motion sensor and the object, i.e., the distance between the motion sensor and you. However, none of your answers is "exact"; there is always some uncertainty, or error, due (for example) to slight differences in how you keep your position steady, your breathing, table vibration, etc. One way to approach such problems is to repeat the measurement several times, but you still must know how to combine the different values to obtain a "best estimate."

As you have probably noticed from your data tables, the chances are particularly good that no two of your values for x are precisely the same. Let us consider a hypothetical experiment in which x was measured five times, with the following values for x:

$$1.015 \text{ m, } 1.021 \text{ m, } 0.994 \text{ m, } 1.017 \text{ m, and } 1.003 \text{ m.}$$

Here the values are given to the nearest millimeter but the variation from value to value is much larger than this. In cases like this, it is better to quote the **average value** $x_{average}$, which is

$$x_{average} = \frac{\sum_{i=1}^{N} x_i}{N} = \frac{\sum_{i=1}^{5} x_i}{5} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} \quad , \qquad N = 5, \qquad (1)$$

where x_1 is the first measured value (=1.015 m), x_2 is the second value, etc., and N is the number of data values, which in this case is equal to five.

Equation (1) provides the answer to the first question posed above. This is how one should combine several measured values to obtain a "best" estimate. In our example above, $x_{average}$ = 1.010 m. Here we have quoted the value of $x_{average}$ to the nearest 0.001 m, since that was the way, the original values were given. However, we have already seen that the spread of results is much larger than 0.001 m, so our value of $x_{average}$ is clearly not reliable or accurate to this level.

This brings us to the second question posed above, which is how to estimate the accuracy of $x_{average}$? To be more precise, the question really is "if we could somehow make a perfect measurement, how much is it likely to differ from $x_{average}$?" The answer is obtained from a quantity called the **sample standard deviation**, which is usually denoted by s_x and in this case, is given by

$$s_x = \sqrt{\frac{\sum_{i=1}^{N}(x_i - x_{ave})^2}{N-1}} = \sqrt{\frac{(x_1 - x_{ave})^2 + (x_2 - x_{ave})^2 + (x_3 - x_{ave})^2 + (x_4 - x_{ave})^2 + (x_5 - x_{ave})^2}{4}} \quad (2)$$

Statistics predicts that for large N values, the chances are about 68% that $x_{average}$ will differ from the exact answer by an amount equal to s_x or less (this assumes that we are dealing with ***random errors***, as we will explain in a moment). It is customary to write our result as

$$x = x_{average} \pm s_x \ (\text{units})$$

In our example above, we have: $x_{average} = 1.010 \text{ m}$ and $s_x = 0.011 \text{ m}$

Therefore, the result should be written as: $x = 1.010 \pm 0.011 \text{ m}$.

There is one important, though obvious, point we should make about the use of s_x from Eq. (2) as an estimate of the experimental uncertainty (experimental error) in x. We have assumed implicitly that there is no "bias" in our measurements. In other words, the probability that a value of x_i will be larger than the perfect value is the same as the probability that it will be smaller than the perfect value. Therefore, the average – Eq. (1) will provide a good estimate of $x_{perfect}$. Errors of this kind are called ***random*** **errors** since their sign (positive or negative) is random.

There is also a second kind of error known as ***systematic errors***. Systematic errors are instrumental or methodological errors causing consistent deviation of results in one direction from the true value. One very typical example of measurement with large systematic error is a scale that was not properly zeroed. That scale would show values that are systematically smaller (or systematically larger) value than the true weight of measured objects.

Another example is if the temperature in the room (and consequently the speed of sound) would be significantly lower than the usual 20°C (= 68°F). Computer needs the exact value of sound speed to properly calculate distance: $x = \Delta t * v_{sound}$. It would then provide values of x_i, which always tend to be larger than $x_{perfect}$, and the average of these x_i values could differ from $x_{perfect}$ by much more than s_x. The only way to cope with systematic errors is to carefully analyze the way your experiment works, to understand how it might be biased and by adjusting equipment. The graphs below illustrate both random and systematic errors.

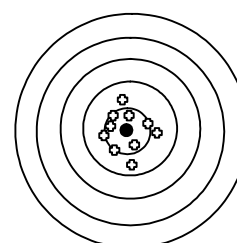Very close agreement between trials, but systematically off the target.

Widespread, random variation of results.
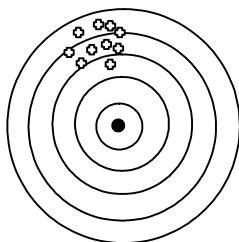
Small random error, but large systematic error.
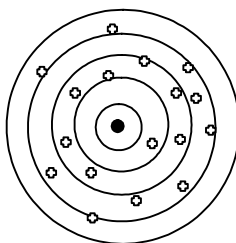
Large random error and small systematic error.

The best situation, i.e., small random error and small systematic error.

It is possible to reduce the <u>systematic error</u> by meticulously designing the experiment and adjusting experimental apparatus (for example, zeroing scales). The random error can be reduced by repeating measurements or by getting better equipment. In case of multiple measurements, we use the **average** value to reduce the random error. Standard deviation provides a good approximation of the random error. However, no matter how good is our apparatus and no matter how many times we repeat measurements **the random error cannot be completely eliminated**. A similar statement is also true for systematic error. Even the most careful calibration would not eliminate the systematic error. Calibration procedure would reduce or minimize the systematic error, but it cannot eliminate the systematic error. The total experimental error depends on the <u>larger</u> od the two types of errors - random or systematic.

Open "M1 Activity 3" by clicking on it. The application program: "*Capstone*" by PASCO Scientific will configure for *Activity 3*.

<u>In this activity,</u> you are going to make multiple distance measurements for an object that have a fixed position using ultrasonic sensor. Since the sensor measures the distance several times per second, we should get many data points in relatively short time. Then you will export data to Excel spreadsheet and calculate the mean (average) value and the standard deviation. It is possible to do using a calculator instead of Excel, but that would be very time consuming. The ultrasonic motion detector makes 10 measurements per second, so after 60 sec of measurements, you should have 600 data points. Entering those manually into a calculator would be very tedious.

Stand in front of the ultrasonic motion sensor exactly 1.00 m away from it. To find the exact position, start the measurements by clicking on the "*Record*" button. For the first 15 seconds motion sensor will show you the numerical value of the distance inside the window "Distance to the sensor". However, it is not going to save any data. Use that initial 15 sec to adjust your position to get distance equal to 1.000 m. After 15 seconds, you should see data points on the position versus time graph. After that moment do not move and do not adjust the position anymore for the next 60 seconds. Try to stand as still as possible but close your eyes after the initial 15 seconds. The motion sensor is quite sensitive, so you will see small variations to the measured distance. After 60 seconds data collection will stop automatically. The complete run takes 15 + 60 = 75 seconds.

Look at the histogram at the bottom of the screen. That histogram shows the number of repeated results (i.e., the frequency of data points). The more often a recorded value fits inside an individual bar (Δx = 0.001 m wide in this case), the higher the bar is. The histogram should resemble a bell-like curve called the normal distribution of experimental data. It is a universal curve that can be applied to various measurements, and it does not depend on the type of measurements. If you make many repeated measurements of the same thing, the results shown as histogram should resemble the normal distribution, bell-shape curve.
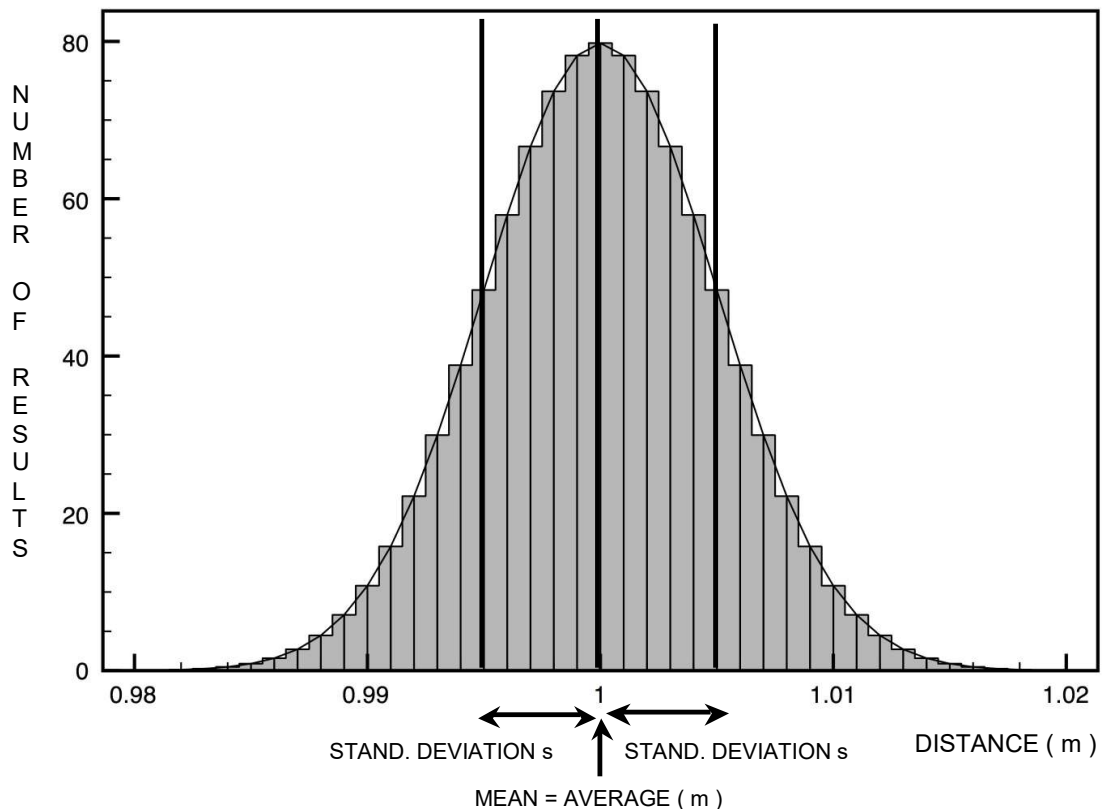
The bell-shaped normal curve is centered at the mean (i.e., the average) value. The width of the bell-shaped curve is proportional to the standard deviation. Narrow curve means small standard deviation, which indicates precise measurements. Standard deviation is commonly used to estimate random, experimental errors. Select "**Print**" from the "**File**" menu. Include this printout to your lab report. Your experimental histogram graph will not look as nice as the ideal one shown above but should resemble it.

Once a good data set is recorded, export data by selecting "**Export Data**" from the "**File**" menu. You should only choose "Position, Ch 1&2" data column for export. Uncheck the markers for other data columns. Save the file to the desktop. Next, transfer the position data to MS Excel for further analysis.

You should have one column of data - "Position, Ch 1&2". With help of MS Excel, calculate the mean (i.e., the average) position and the standard deviation of position using equations (1) and (2). Keep in mind that the total number of measurements is now equal to N = 600. Record those numbers on the data sheets. Manually draw the lines indicating the average value and the average +/- standard deviation on the "Histogram" printout. Microsoft Excel offers multiple functions for different versions of standard deviation (some of these are used in statistics). **Make sure to use "STDEV" function - standard deviation based on a sample.**

Make sure to complete the following tasks while in the lab room:

You must submit the answers to the prelaboratory questions online.   (3.5 points)

1. Two graphs from Activities 1 and 2.   (2 × 1 = 2 points)

2. One histogram printed during Activity 3.   (1 point)
    (Write the title as well as student names and on each graph.)

3. Your completed Data Sheets.   (3.5 points)

4. **Return** the completed lab report to your lab TA.